# II. *Researches towards establishing a Theory of the Dispersion of Light. No. III.*

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Received October 20, 1836—Read January 19, 1837.

## *Introductory Remarks.*

IN two former portions of researches on the subject of dispersion *, I have discussed all the observed refractive indices for definite rays, in different media, which had come to my knowledge; consisting of those for ten media determined by FRAUNHOFER, and those in ten other cases by M. RUDBERG, comparing them with the calculated results of the theory of M. CAUCHY; and the agreement is sufficiently close. In those papers, and elsewhere, I have remarked the importance of extending the inquiry, especially to media of higher dispersive power; in which cases (as appears from the nature of the formula) the theory would be put to a more precise test.

In the former instances the work of determining the indices was done to my hands, and I could proceed to the theoretical computations with the most perfect confidence in the accuracy of experimental data, furnished from the labours of observers so well known for precision and skill, and obtained, too, before the formula of theory had been deduced.

In any comparison of theory with experiment, it is, in all points of view, far more satisfactory that such comparison should be made with the observations of others rather than those of the theoretical computer himself. In the present instance, however, this desirable condition has not been fulfilled. Though the importance of obtaining a series of indices for the standard rays in different media had been long since pointed out and acknowledged by the most eminent philosophers, yet no observer was found to undertake the task of carrying on the work which FRAUNHOFER and RUDBERG had so successfully begun. I was thus left to make an attempt myself to supply the deficiency; and my observations were communicated, and printed copies distributed, to the Physical section of the British Association at the Bristol meeting in August 1836†.

From the remarks prefixed to those results, the scientific reader will, I trust, be sufficiently enabled to judge of the nature and degree of accuracy of the observations.

* Philosophical Transactions, 1835, Part I.; and 1836, Part I.

† This tract now forms one of the series of memoirs published by the Oxford Ashmolean Society.

I conceive, in general, the indices deduced may be relied on as exact, to at least three places of decimals. Some exceptions are noticed in the tract as less certain, viz. the oils of Angelica, Cummin, and Pimento, and the balsam of Peru. Such as they are, however, these results form (as far as I am aware) the only existing data for pursuing the comparison with theory. But I trust they may not be thought insufficient, when we consider that in the present stage of the inquiry the object to be aimed at seems chiefly such a general comparison as may enable us to see whether the main principle of the undulatory explanation of the dispersion is applicable, with a sufficient approach to precision, to encourage us to pursue the theory, or whether it must be abandoned, and some new principle sought.

With respect to media of low dispersive power, little doubt can exist. I have therefore not thought it worth while to go through the calculations for many of this class, but have confined my examination in the present instance chiefly to the higher cases to which my observations have extended.

*Method of Calculation.*

It may be necessary to premise a notice of the method of calculation adopted in the present communication, as it differs from that employed in my two former papers. That method consisted in finding, in the first instance, by a tentative process, (virtually equivalent to assuming the two extreme indices from observation,) the fundamental arc $\theta$, from which the others were derived on dividing by $\lambda$ for each ray, so as to fulfill the conditions of the approximate formula

$$\frac{1}{\mu} = \mathrm{H}\left\{\frac{\sin\left(\frac{\theta}{\lambda}\right)}{\left(\frac{\theta}{\lambda}\right)}\right\}.$$

It also appears from the investigations given in several consecutive papers in the London and Edinburgh Journal of Science, &c., that this formula is in fact obtained by supposing the sum of a series of analogous terms collected into one, with a common constant coefficient H. This simplified hypothesis (from the accordances already obtained) is evidently very near the truth for the whole range of media hitherto examined.

Professor Sir W. R. Hamilton afterwards pointed out (besides a direct process for performing this approximate calculation) a method of investigating the exact expression when we do not allow the above assumption as to the coefficients, viz.

$$\left(\frac{1}{\mu}\right)^2 = \mathrm{S}\left\{\mathrm{H}^2\left\{\frac{\sin\left(\frac{\theta}{\lambda}\right)}{\left(\frac{\theta}{\lambda}\right)}\right\}^2\right\}.$$

This is explained at large in the journal just named*; and in a subsequent number†

* March 1836. † August 1836.

I have stated some further particulars illustrative of the process. It is this method which I shall use in the present investigation, and therefore here merely quote the resulting formula from the last-mentioned paper. It is as follows;

$$\mu_\iota - \mu_F = a_\iota (\mu_F - \mu_B) + b_\iota (\mu_H - 2\,\mu_F + \mu_B) :$$

where the three indices of refraction are assumed from observation for the respective rays B, F, and H, in the particular medium; while $\mu_\iota$ is the index sought, any other of the four remaining of the seven standard rays, corresponding to which $a_\iota$ and $b_\iota$ are constants for the ray, independent of the particular medium, and which have been found from the values of $\lambda$ in the paper last referred to, as follows:

$$\log a_C = \bar{1}\cdot95433 \quad \log b_C = \bar{2}\cdot65253$$

$$\log a_D = \bar{1}\cdot80441 \quad \log b_D = \bar{1}\cdot06281$$

$$\log a_E = \bar{1}\cdot49646 \quad \log b_E = \bar{1}\cdot03196$$

$$\log a_G = \bar{1}\cdot74027^{*} \quad \log b_G = \bar{1}\cdot62954^{*}.$$

The nature of the process of computation is obvious from an inspection of the formula; and the data here stated will suffice for those who may wish to verify the calculations. I proceed to give the results in a tabular form, and then to offer such general conclusions as I think may be safely derived from them.

*Comparison of Refractive Indices from* CAUCHY'S *Theory and from observation.*

1. Nitric Acid.

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·3988 | | |
| C | 1·3998 | 1·3998 | ·0000 |
| D | 1·4026 | 1·4024 | — ·0002 |
| E | 1·4062 | 1·4058 | — ·0004 |
| F | 1·4092 | | |
| G | 1·4155 | 1·4156 | + ·0001 |
| H | 1·4211 | | |

2. Muriatic Acid.

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·4050 | | |
| C | 1·4065 | 1·4061 | — ·0004 |
| D | 1·4095 | 1·4089 | — ·0006 |
| E | 1·4130 | 1·4125 | — ·0005 |
| F | 1·4160 | | |
| G | 1·4217 | 1·4222 | + ·0005 |
| H | 1·4265 | | |

3. Sulphuric Acid.

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·4321 | | |
| C | 1·4329 | 1·4329 | ·0000 |
| D | 1·4351 | 1·4349 | — ·0002 |
| E | 1·4380 | 1·4375 | — ·0005 |
| F | 1·4400 | | |
| G | 1·4440 | 1·4448 | + ·0008 |
| H | 1·4468 | | |

4. Oil Angelica.

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·4836 | | |
| C | 1·4863 | 1·4849 | — ·0014 |
| D | 1·4887 | 1·4882 | — ·0005 |
| E | 1·4932 | 1·4924 | — ·0008 |
| F | 1·4963 | | |
| G | 1·5049 | 1·5035 | — ·0014 |
| H | 1·5099 | | |

* The two last numbers include the correction of an error which I detected in those given in the paper referred to.

**5. Oil Cummin.**

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·5023 | | |
| C | 1·5043 | 1·5039 | — ·0004 |
| D | 1·5075 | 1·5079 | + ·0004 |
| E | 1·5130 | 1·5136 | + ·0006 |
| F | 1·5196 | | |
| G | 1·5326 | 1·5317 | — ·0009 |
| H | 1·5432 | | |

**6. Oil Sassafras.**

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·5252 | | |
| C | 1·5267 | 1·5268 | — ·0001 |
| D | 1·5312 | 1·5315 | + ·0003 |
| E | 1·5380 | 1·5376 | — ·0004 |
| F | 1·5441 | | |
| G | 1·5568 | 1·5569 | + ·0001 |
| H | 1·5687 | | |

**7. Oil Pimento.**

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·5281 | | |
| C | 1·5317 | 1·5300 | — ·0017 |
| D | 1·5347 | 1·5348 | + ·0001 |
| E | 1·5422 | 1·5412 | — ·0010 |
| F | 1·5478 | | |
| G | 1·5599 | 1·5605 | + ·0006 |
| H | 1·5719 | | |

**8. Kreosote.**

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·5327 | | |
| C | 1·5341 | 1·5344 | + ·0003 |
| D | 1·5387 | 1·5392 | + ·0005 |
| E | 1·5457 | 1·5455 | — ·0002 |
| F | 1·5520 | | |
| G | 1·5645 | 1·5646 | + ·0001 |
| H | 1·5762 | | |

**9. Balsam of Peru.**

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·5846 | | |
| C | 1·5869 | 1·5869 | ·0000 |
| D | 1·5931 | 1·5935 | + ·0004 |
| E | 1·6029 | 1·6028 | — ·0001 |
| F | 1·6130 | | |
| G | 1·6336 | 1·6336 | ·0000 |
| H | 1·6533 | | |

**10. Oil of Anise. Temp. 21°·2.**

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·5452 | | |
| C | 1·5473 | 1·5473 | ·0000 |
| D | 1·5533 | 1·5533 | ·0000 |
| E | 1·5623 | 1·5615 | — ·0008 |
| F | 1·5705 | | |
| G | 1·5883 | 1·5887 | + ·0004 |
| H | 1·6061 | | |

**11. Oil of Anise. General Mean.**

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·5469 | | |
| C | 1·5489 | 1·5490 | + ·0001 |
| D | 1·5553 | 1·5551 | — ·0002 |
| E | 1·5642 | 1·5635 | — ·0007 |
| F | 1·5726 | | |
| G | 1·5904 | 1·5909 | + ·0005 |
| H | 1·6082 | | |

**12. Oil of Anise. Temp. 15°·2.**

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·5486 | | |
| C | 1·5506 | 1·5509 | + ·0003 |
| D | 1·5574 | 1·5570 | — ·0004 |
| E | 1·5661 | 1·5656 | — ·0005 |
| F | 1·5747 | | |
| G | 1·5926 | 1·5932 | + ·0006 |
| H | 1·6103 | | |

**13. Sulphuret of Carbon. Temp. 22°.**

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·6145 | | |
| C | 1·6176 | 1·6178 | + ·0002 |
| D | 1·6272 | 1·6269 | — ·0003 |
| E | 1·6405 | 1·6392 | — ·0013 |
| F | 1·6521 | | |
| G | 1·6763 | 1·6776 | + ·0013 |
| H | 1·7009 | | |

**14. Sulphuret of Carbon. Temp. 12°.**

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·6234 | | |
| C | 1·6258 | 1·6268 | + ·0010 |
| D | 1·6369 | 1·6361 | — ·0008 |
| E | 1·6496 | 1·6485 | — ·0011 |
| F | 1·6616 | | |
| G | 1·6864 | 1·6870 | + ·0006 |
| H | 1·7103 | | |

**15. Oil of Cassia.**

| Ray. | Index observed. | Index calculated. | Difference. |
|---|---|---|---|
| B | 1·5885 | | |
| C | 1·5918 | 1·5913 | — ·0005 |
| D | 1·6017 | 1·6000 | — ·0017 |
| E | 1·6155 | 1·6135 | — ·0020 |
| F | 1·6295 | | |
| G | 1·6607 | 1·6646 | + ·0039 |
| H | 1·7002 | | |

### *General Conclusions.*

With regard to some of the first media above compared, the agreement of observation and theory is sufficiently near; but they are low in the scale of dispersion. The cases before mentioned as being somewhat uncertain do not allow of a close comparison. The balsam of Peru (though only from approximate data) and the kreosote, present higher numbers, and the agreement is therefore the most satisfactory. This is also the case with oil of anise seed, though perhaps not to the same degree; but the differences are probably not greater than what may reasonably be allowed.

In all these cases, however, we trace some uniformity in the character of the differences; theory being always in defect for the ray E, and in excess for G. When we proceed to the sulphuret of carbon this becomes more apparent; and the differences increase for the more refrangible rays. And lastly, in oil of cassia the same regular order of deviation is more marked, and the differences much greater, especially in the rays E and G. This regularity of character, as well as increase in the amount of the discrepancy, at once shows that it is at least partly due to some other cause than errors of observation. Yet we must remark, that even in these cases there is a sort of general accordance preserved between observation and theory, the agreement being still accurate to the second place of decimals.

The following considerations, then, suggest themselves:

1. It is to be observed that in all these cases closer accordances might be obtained if we took slightly different values for the assumed indices of B, F, and H; such as would be consistent with the probable errors of observation. In oil of cassia, however, I have found that the errors, even when thus distributed among all the indices, are still too great.

2. The constants $a$ and $b$ are derived solely from the values of $\lambda$ for each ray, taken from the well-known determinations of Fraunhofer from interference. It is doubtless possible that these determinations may be affected by errors. In computing, by the method here used, some of Fraunhofer's indices, in which small discrepancies were found, Sir W. R. Hamilton undertook to investigate what amount of alteration in the values of $\lambda$ would account for those differences. He communicated his researches in a letter to myself, with the values of $\lambda$, and consequently those of $a$ and $b$, thus altered. I have repeated the calculation for oil of cassia, but find this change quite insufficient to remove the discrepancy; and that, in fact, a much larger alteration than could for a moment be allowed in Fraunhofer's data, must be supposed, in order to produce any sensible effect.

3. The entire method of computation here followed, though founded on the exclusion of those approximate suppositions which are allowed in the simpler formula, is yet dependent on the omission of terms in the series for $\mu$ beyond the three first*.

* See London and Edinburgh Journal of Science, &c., March 1836, eq. 9 and 13.

Since in this analysis we know nothing of the values of the coefficients of this series, but proceed solely by the elimination of them, it is still possible that in the neglected terms we may find the means of explaining the observed discrepancy. In a word, it remains an important subject for future research whether a further prosecution of the analysis, either by this method or by that which M. CAUCHY has himself recently pointed out, or that of Mr. KELLAND, may not lead to more successful results.

For the present I will only remark, by way of recapitulation, that upon the whole I conceive the formula, as already deduced from the undulatory theory, applies sufficiently well to the case of media whose dispersion is as high as that of oil of anise seed. It also represents, with a certain general approximation to the truth, the indices of some more highly dispersive bodies. It is therefore extremely probable that the essential principle of the theory has some real foundation in nature. While, looking at the regularity of the deviation, it seems likely that the formula only requires to receive some further development or extension, in order to make it apply accurately to the higher cases, while it shall still include the simpler form which so well accords with the lower.

*October* 10, 1836.